Exercise 289

For the following exercises, solve the logarithmic equation exactly, if possible.

$$\log_6(x+9) + \log_6 x = 2$$

Solution

Combine the logarithms.

$$\log_6(x+9)x = 2$$

The base is 6, the exponent is 2, and the result is $(x+9)x$.

$$6^2 = (x+9)x$$

Use the distributive law on the right side.

$$36 = x^2 + 9x$$

Bring all terms to one side.

$$x^2 + 9x - 36 = 0$$

Factor the left side by looking for two numbers that multiply to -36 and add to $+9$: 12 and -3.

$$(x+12)(x-3) = 0$$

Use the zero product property: If two numbers multiply to zero, then one or both of the numbers must be zero.

$$x + 12 = 0 \quad \text{or} \quad x - 3 = 0$$

$$x = -12 \quad \text{or} \quad x = 3$$

Looking at the original equation, plugging in $x = -12$ doesn't work because the logarithm of a negative number is undefined. Therefore,

$$x = 3.$$